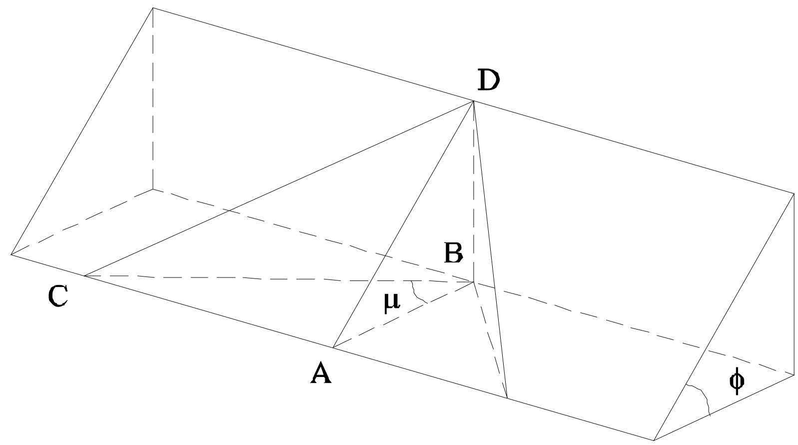
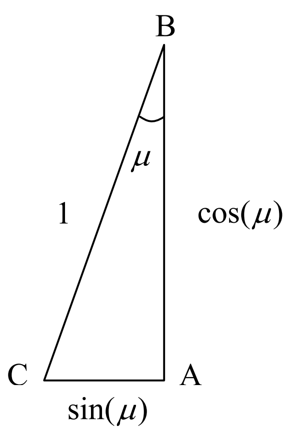
An easy way to visualize the problem of cutting crown molding is to consider the problem of cutting a miter on a right triangular prism. In order to cut the prism such that it bends around a corner, one can set the miter angle to half of the angle of bend.

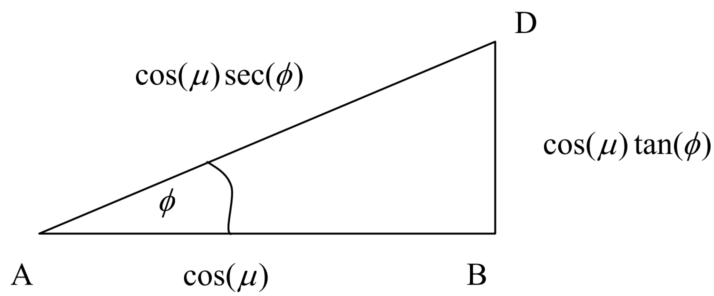


The following trig identities will be useful throughout the derivation:

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$



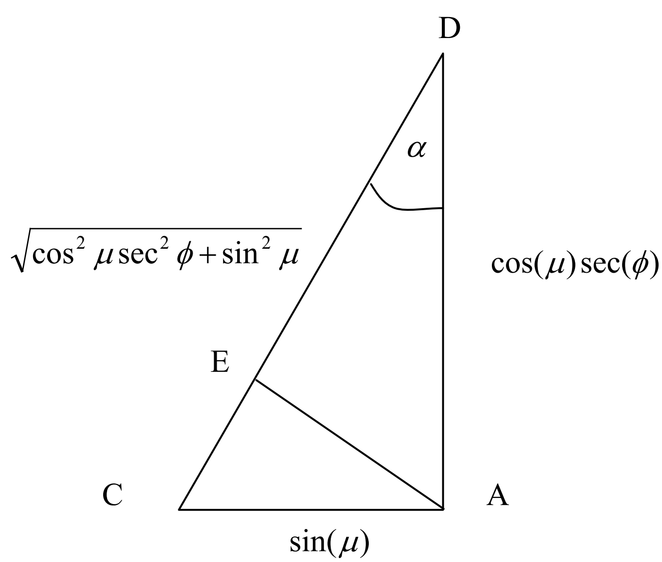
The solid of interest consists of 4 triangular faces: *ABC, ABD, ADC*, and *CBD*. Without loss of generality we set the hypotenuse of *ABC* to 1. The flat miter angle is denoted μ and is the angle between line segments *BC* and *BA*. By definition, the line segment $CA = \sin(\mu)$ and $BA = \cos(\mu)$. We now turn our attention to the triangle *ABD*. We wish to solve for the length of the line segments *BD* and *AD* given that the angle between *AD* and AB is denoted by ϕ, which we have chosen to represent the crown angle. The following figure shows the result:



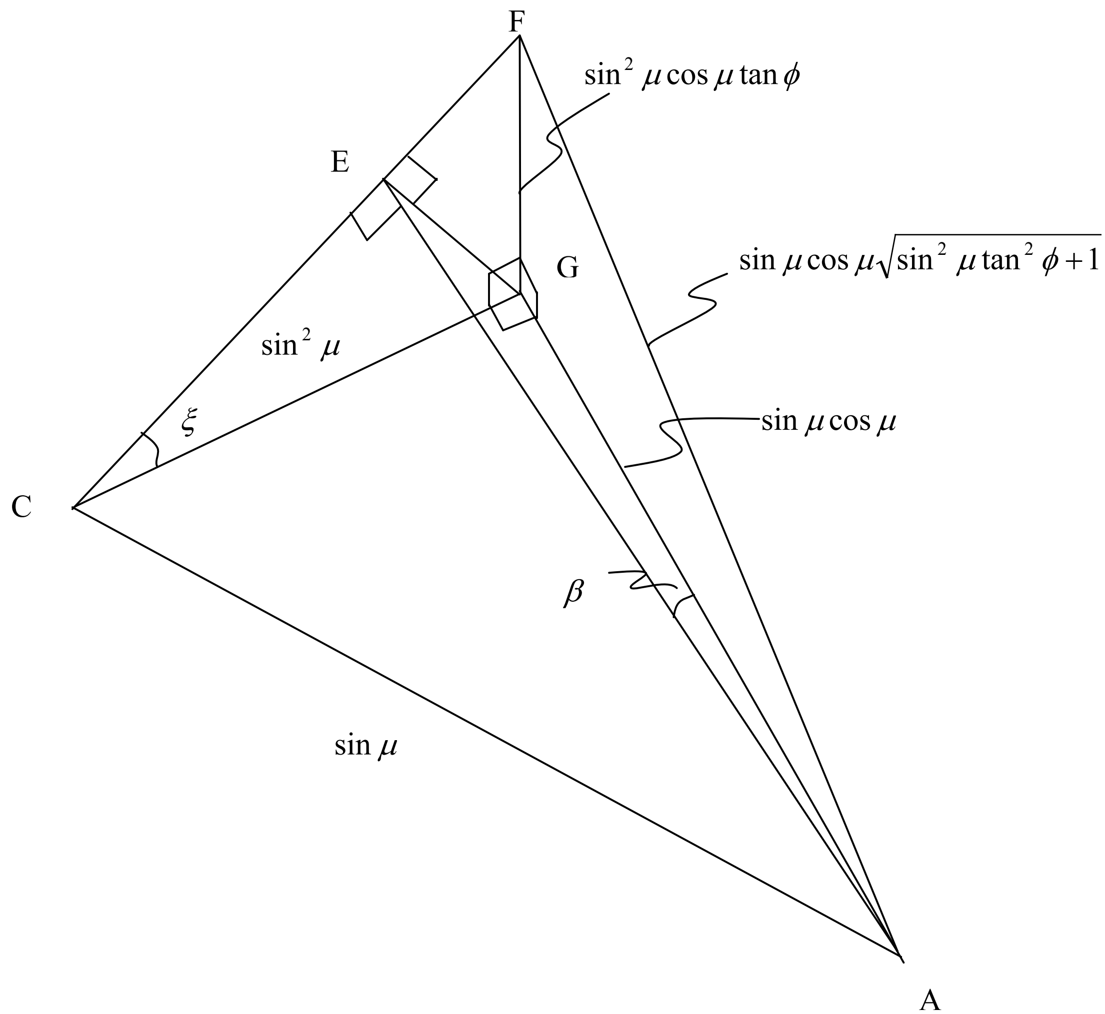
Next we analyze triangle *ACD*. The angle between the line segments *AD* and *CD* is the miter gage angle α which we would like to express in terms of the flat miter angle μ and crown angle ϕ. Note that two sides of this triangle have already been calculated, i.e. $AD = \cos(\mu)\sec(\phi)$ and $AC = \sin(\mu)$. Since AC is opposite α and AD is adjacent to α, we find that:

$$\tan\alpha = \frac{AC}{AD} = \frac{\sin\mu}{\cos\mu\sec\phi} = \tan\mu\cos\phi$$

which is the well known formula for the miter angle as a function of the flat miter angle and crown angle.. We will now complete the solution of triangle *ACD* by using the Pythagorean theorem to compute the length of the hypotenuse.



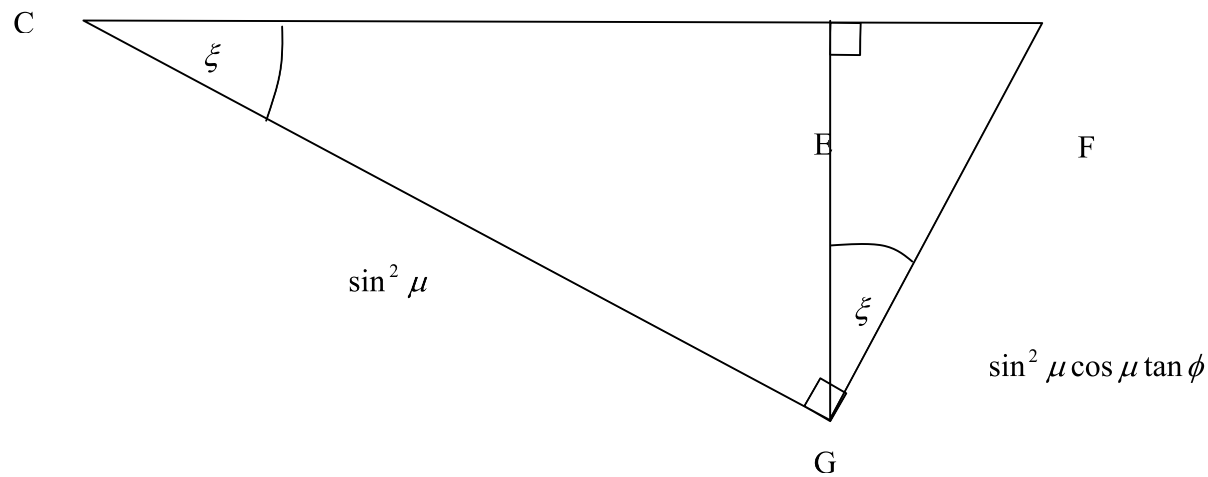
We now define a new triangle, *AEG* with $AE \perp DC$ and $EG \perp DC$ and the angle between *AE* and *AG* is the saw bevel angle which is denoted β. A point *F* is defined by the intersection of *CD* and a line perpendicular to *CB* passing through the point *G*. Note that *AGC* and *ABC* are geometrically similar as are *CGF* and *CBD*. Recall that $BC = 1$, $AC = \sin\mu$ and $DB = \cos\mu\tan\phi$, thus it follows that $CG = \sin^2\mu$, $FG = \sin^2\mu\cos\mu\tan\phi$ and $AG = \sin\mu\cos\mu$.



Now AF can be solved using the Pythagorean theorem:

$$AF = \sqrt{\sin^4 \mu \cos^2 \mu \tan^2 \phi + \sin^2 \mu \cos^2 \mu} = \sin \mu \cos \mu \sqrt{\sin^2 \mu \tan^2 \phi + 1}$$

Now note that CGE and EGF are geometrically similar.



By the Pythagorean theorem we compute:

$$CF = \sqrt{\sin^4 \mu + \sin^4 \mu \cos^2 \mu \tan^2 \phi} = \sin^2 \mu \sqrt{1 + \cos^2 \mu \tan^2 \phi}$$

We define the angle between *CG* and CF as ξ and we can compute

$$\sin \xi = \frac{\sin^2 \mu \cos \mu \tan \phi}{\sin^2 \mu \sqrt{1 + \cos^2 \mu \tan^2 \phi}} = \frac{\cos \mu \tan \phi}{\sqrt{1 + \cos^2 \mu \tan^2 \phi}}$$

and

$$\cos \xi = \frac{\sin^2 \mu}{\sin^2 \mu \sqrt{1 + \cos^2 \mu \tan^2 \phi}} = \frac{1}{\sqrt{1 + \cos^2 \mu \tan^2 \phi}}$$

Now $EG = \cos \xi \sin^2 \mu \cos \mu \tan \phi$ or

$$EG = \frac{\sin^2 \mu \cos \mu \tan \varphi}{\sqrt{1 + \cos^2 \mu \tan^2 \phi}}$$

Now by the Pythagorean theorem,

$$EA = \sqrt{\frac{\sin^4 \mu \cos^2 \mu \tan^2 \varphi + \cos^2 \mu \sin^2 \mu (1 + \cos^2 \mu \tan^2 \phi)}{1 + \cos^2 \mu \tan^2 \phi}}$$

or

$$EA = \sin \mu \cos \mu \sqrt{\frac{\sin^2 \mu \tan^2 \phi + 1 + \cos^2 \mu \tan^2 \phi}{1 + \cos^2 \mu \tan^2 \phi}} = \sin \mu \cos \mu \sqrt{\frac{\tan^2 \phi (\sin^2 \mu + \cos^2 \mu) + 1}{1 + \cos^2 \mu \tan^2 \phi}}$$

which further simplifies to:

$$EA = \sin \mu \cos \mu \sqrt{\frac{\sec^2 \phi}{1 + \cos^2 \mu \tan^2 \phi}}$$

finally:

$$EA = \sin \mu \cos \mu \sec \phi \sqrt{\frac{1}{1 + \cos^2 \mu \tan^2 \phi}}$$

We may now compute the sine of the saw bevel angle β as:

$$\sin \beta = \frac{EG}{EA} = \frac{\sqrt{1 + \cos^2 \mu \tan^2 \phi}}{\sin \mu \cos \mu \sec \phi} \bullet \frac{\sin^2 \mu \cos \mu \tan \phi}{\sqrt{1 + \cos^2 \mu \tan^2 \phi}}$$

simplifying we obtain:

$$\sin \beta = \frac{\sin \mu \tan \phi}{\sec \phi}$$

further simplification yields the well known formula for the saw bevel angle as a function of the flat miter angle and crown angle:

$$\sin \beta = \sin \mu \sin \phi$$

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